

# The maximal subgroups of $F_4$ , $E_6$ and ${}^2E_6$ and related almost simple groups, Errata

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22nd April, 2025

I distinguish four types of corrections, in order of increasing seriousness:

- (Extra) Additional information that was not available at the time of writing, or that I did not know about.
- (Improve) Typographical issues, where what is written is still correct, but there is a nicer way of phrasing it, or I could choose a better symbol.
- (Typo) Typographical errors, where I have spelled a word wrongly, used the wrong symbol, and so on.
- (Error) Errors in proofs or statements.

When I give each correction, I will label it with one of these monikers.

- (i) (Error) In Table 8 there is a torus normalizer with structure  $(q^2 + 1)^2 \cdot (4 \circ \mathrm{GL}_2(3))$ . The group  $4 \circ \mathrm{GL}_2(3)$  is not a subgroup of the Weyl group of type  $F_4$ , and the correct group should be  $\mathrm{SL}_2(3) \rtimes 4$ , which is the centralizer of an element of order 4 in  $W(F_4)$ . The original paper of Liebeck–Saxl–Seitz also contains this error, and I copied their tables over without double-checking.

Thanks to Mikko Korhonen for noticing this.

- (ii) (Error/Typo) In Table 8 the large Ree subgroups have a badly explained structure. I have written  ${}^2F_4(q_0)$  in analogy with the fact that  $\mathrm{SU}_n(q) \leq \mathrm{SL}_n(q^2)$ . This doesn't really work for large Ree groups, and the more standard notation is that  ${}^2F_4(2) \leq F_4(2)$ , for example. So this line should read that  ${}^2F_4(q) \leq F_4(q)$ .

Thanks to Tim Burness for mentioning this.

More importantly, when writing this erratum I noticed that the condition that  $q$  is an odd power of 2 is missing from this, but since there is no such group as  ${}^2F_4(4)$  this is heavily implied, even if it is an error that it is not present.

(iii) (Error) In Table 9 the subgroup  $d^2.(P\Omega_8^+(q) \times ((q-1)/d)^2/e).d^2.\text{Sym}(3)$  is correct. Comparing this with Table 10, we find the correct version should be  $d^2.(P\Omega_8^+(q) \times ((q+1)/d)^2/e').d^2.\text{Sym}(3)$ , not the  $d^2.(P\Omega_8^+(q) \times ((q+1)/d)^2).d^2.\text{Sym}(3)$  as given.

Thanks to Melissa Lee for noticing this.

(iv) (Error) In Table 9 the novelty subgroup should be  $[q^{31}].(\text{SL}_2(q) \times \text{SL}_2(q) \times \text{SL}_3(q)).(q-1)^2/e$ . There's a missing  $(q-1)$  factor in the torus (obvious) and the action when removing the centre is to quotient by a diagonal  $e$  in the  $\text{SL}_3(q) \times (q-1)^2$  subgroup. This yields a central product, so we can write the structure above.

Thanks to Tim Burness for highlighting this.